Optimal Shotlines for Lethality Testing

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Abstract

A method for choosing shotline parameters for use in the experimental determination of lethal areas is presented. A criterion for an optimal shotline is proposed. A method for implementing this criterion for determining the optimal shotline in a multidimensional shotline parameter space is developed and implemented. The method utilizes both random shotline points in the multidimensional field of concern, while also biasing the choice of the optimal shotline to conform to the criterion. Examples are presented to show the application of the method for a variety of cases. This progressive use of optimal data points (or N-dimensional shotlines) can be used to recursively improve the description of the lethal volume using an optimizing strategy, such as simulated annealing, to optimize the development of the lethality model.

Introduction

The basis for efficient lethality testing is to obtain the maximum amount of definition of the lethal area with the minimum number of lethality tests. Lethality tests can refer to computer simulations as well as physical experiments. A previous paper, Reference 1, showed how to develop the definition of a lethal area with a relatively few number of data points using the "simulated annealing" optimization process. The points, in that paper, were chosen apriori as a representation of typical points within the lethal area. A more efficient way of picking the data points, or N-dimensional shotlines (where there are N shotline parameters – hit point coordinates, strike angle, velocity, etc.), would be to use the developing description of the lethal area to point toward the next shotline. Thus the succeeding data points would bootstrap each other by using the increasingly accurate definition of the lethal area to choose a succeeding data point that would provide the maximum increase in lethal area definition. This is especially important if N is larger than three, since the number of required data points starts to become very large and visualization of the lethal volume becomes complex.

Since the contours of the lethal area are unknown at the beginning of the data collection, a distribution of the data points is required within an area that encompasses the maximum area that any lethal impact could feasibly occur. Initially this area could be predicted as too large or too small, and require modification later. The easiest distribution to cover such an area is a uniform distribution. A uniform distribution is unbiased within the given bounds and, for enough trials, insures that no part of the lethal area will be overlooked. The problem with a uniform distribution is that it doesn't minimize the number of data points, usually requiring a large number.

A better alternative is to use a biased sampling method. This is required if the number of data points are to be minimized. Generally, to minimize the number of data points, we would want to confine the data points to the most diagnostic points on the lethal volume. If the lethal volume is sharply defined by a boundary, then the most diagnostic points are around and close to the boundary. When someone is handpicking points to define a sharp lethal volume boundary, they tend naturally to choose points around the apparent boundary.

If the lethal volume is not sharply defined, it is difficult to handpick the most diagnostic data points and a more automatic criterion is required. This is especially true when N is greater than three. A criterion for the optimal shotline is proposed as a point in the N dimensional shotline parameter space where the lethality is changing the fastest with regard to the N coordinates. Analytically this would mean maximizing dF, where F is the lethality function and,

$$dF = \sum_{i=1}^{N} \frac{\partial F}{\partial x_i} \quad ,$$

where the x_i are the coordinates
Determining the point(s) where this is true can be a daunting task, even when dealing with an analytic function. The optimizing algorithm that is proposed will handle analytical and non-

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Abstract

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analytical determinations of F, for any multidimensional shotline space, as long as, given a set of coordinates, F can be defined for those coordinates.

Optimizing algorithm

The process of implementing the optimizing algorithm method is composed of the following steps:

1. A point is uniformly randomly picked in the N dimensional space that is bounded by the points $minx_i$ and $maxx_i$ for all the N x_i . These boundaries represent the expected limits of the N parameters of the shotlines.
2. After this first point, X_0, is picked, a large number (say, 20 to 100) of other points, X_j, are also uniformly randomly picked.
3. The slope, S_j, of the X_j points to X_0 is calculated by:

$$S_j = Abs\left(\frac{\left(F_0 - F_j\right)}{\sqrt{\sum_{i=1}^{N}\left(\left(X_0\right)_i - \left(X_j\right)_i\right)^2}}\right)$$

4. The point X_1 is chosen as the X_j corresponding to the largest S_j.
5. The midpoint between X_0 and X_1 (X_m) is chosen , where:

$$\left(X_m\right)_i = \frac{\left(X_0 + X_1\right)_i}{2}$$

6. The slope between X_m and both X_0 and X_1 is calculated as in step 3.
7. If the larger slope is between X_0 and X_m, then X_m becomes X_1. Otherwise, X_m becomes X_0.
8. The new X_0 and X_1 are run through steps 5, 6, and 7 for a defined number (10-50) of times.
9. The final X_m is the estimated optimum shotline.

Application of the algorithm to simple boundaries

The following examples are applications of the algorithm to a lethality function that is a function of a single parameter, x.

Consider Figure 1. where the lethality function is a step function (blue line) and changes discontinuously from 1 to 0 at some value of x along the abscissa. Only one application of the algorithm is shown. The green points are the intermediate points leading to the estimated optimum point (red point). The dotted vertical line represents the average position of the optimum point for a large number of applications of the algorithm. As can be seen, the algorithm easily pinpoints the edge of a step function.

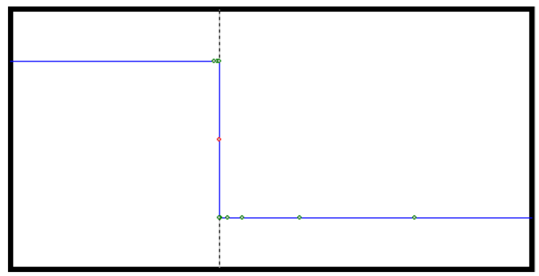

Figure 1. Optimum Point on Step Function

Figure 2. shows a similar situation for the case where the lethality function has a linear transition from its maximum value to its minimum value. In this case there is no single point that represents the maximum change in the curve with x. The application of the algorithm for this one particular case gives an optimal point near the bottom end of the linear slope, but the average position of the optimal point for a large number of applications of the algorithm is near the center of the slope.

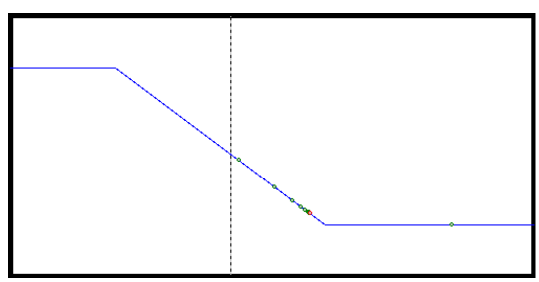

Figure 2. Optimal Point on Linear Function

Figure 3. shows a lethality function with an inflection point, which is where the function is changing the fastest. The average position of the optimum point is very close to the inflection point.

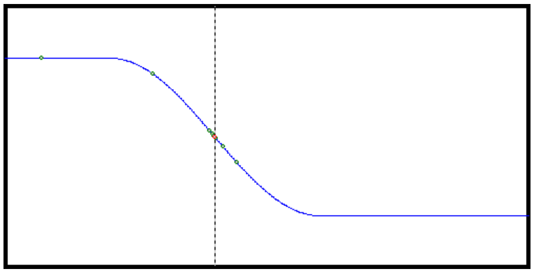

Figure 3. Optimal Point on Function with Inflection Point

Figures 4.-6. show the optimum point for lethality functions, which decrease in slope with their distance from the left edge of the transition. The optimum points are close to this left edge.

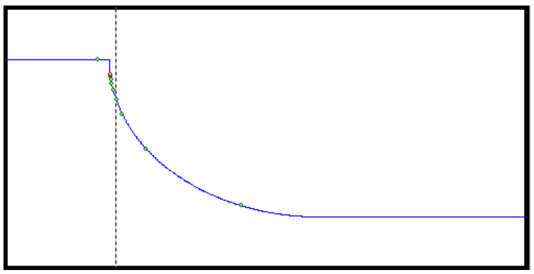

Figure 4. Optimal Point for Function with Strongly Decreasing Slope

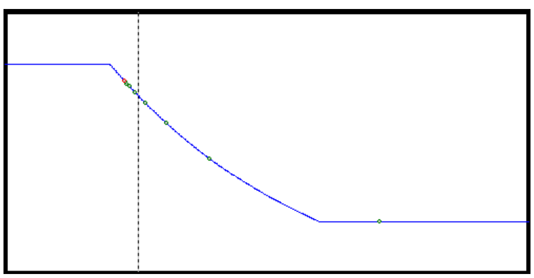

Figure 5. Optimal Point for Function with Slowly Decreasing Slope

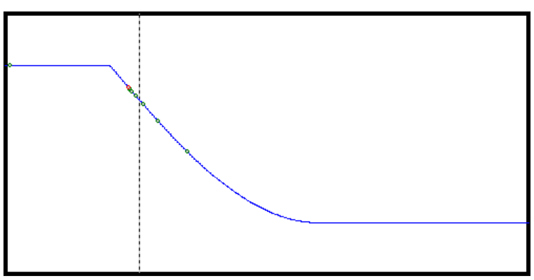

Figure 6. Optimal Point for Function with Linear to Decreasing Slope

Figure 7. is the opposite case where the slope increases as it approaches the right edge of the transition. In this case the optimum point is close to that right edge.

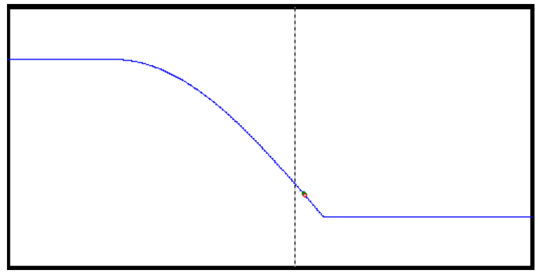

Figure 7. Optimal Point for Function with Increasing Slope

These results show that the algorithm will produce an optimal point near (or at) the place where the lethality function is changing the fastest with the co-ordinate, x. Even where there is a linear transition, the algorithm chooses this transition.

Application of the algorithm to more complex boundaries

The previous Figures illustrate how the algorithm handles various types of 1-parameter boundaries or transitions to find an optimal point. The following Figures show the operation of the algorithm on more complex lethality functions (2-parameter boundaries) where the optimum points form a set of points that define a locus of optimal shotlines. The green points are the uniformly random starting points and the red points are the estimated optimum shotline points.

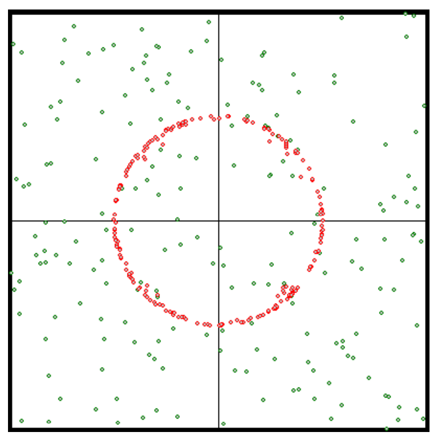

Figure 8. Set of Optimal Points for Spherical Lethality Function

Consider Figure 8., where the lethality function is a sphere, appearing as a circle in the shotline parameter space. The lethality is the spherical height for a shotline whose parameters are inside the circle and the lethality=0, for any other shotline. Here the lethality is a function of two

parameters. The locus of the optimum shotline points is the periphery of the circle.

In Figure 9., the lethality function is a binormal function of two parameters. The function is offset from the origin along the abscissa. The locus of optimum shotline points is an ellipse with a semi major axis equal to the standard deviation of the binormal along the abscissa and a semi minor axis equal to the standard deviation of the binormal along the ordinate axis. The maximum slope for a binormal function occurs at these points. An interesting feature of the locus is that there are fewer points near the left edge of the parameter space. This is because there are fewer random points in that vicinity to generate the optimum shotline points there.

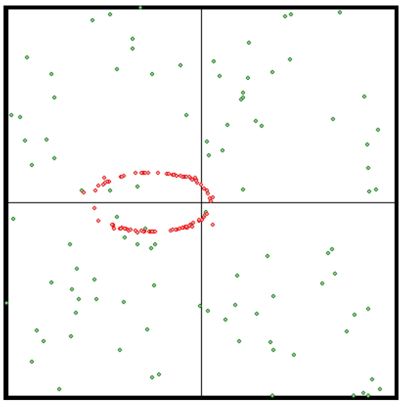

Figure 9. Set of Optimal Points for Binormal Lethality Function

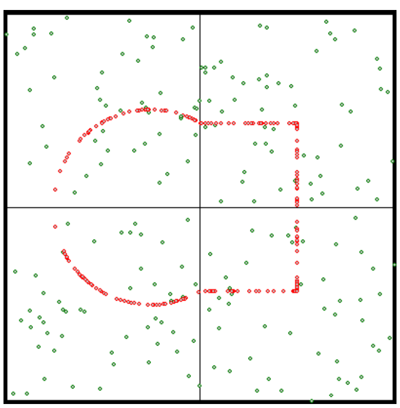

Figure 10. Set of Optimal Points for Compound Lethality Function

Figure 10 shows that more complex lethality functions made up of simple shapes, in this case a cylinder fused to a parallelepiped, can have the optimum shotline point locus determined by the algorithm.

Figure 11 shows the locus of optimum shotline points for two separated lethality functions (cylinders). Any number of such separated functions would also have their optimum shotline loci revealed by the algorithm. This shows the advantage of an algorithm that combines a uniform random search together with a bias towards the nearest lethality boundary.

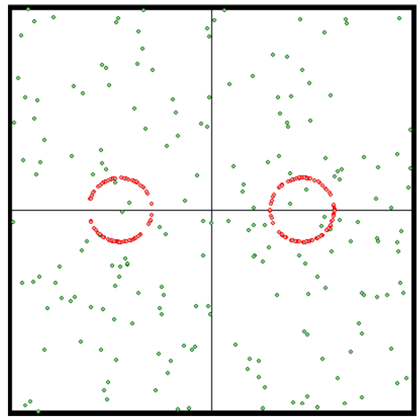

Figure 11. Set of Optimal Points for Two Separated Cylindrical Lethality Functions

<u>Application of the algorithm to N parameters</u>

The preceding examples utilized 1 and 2 dimensional parameters to illustrate the results of applying the algorithm. In reality, the algorithm is most useful for the cases where there are more than 3 shotline parameters. It is difficult to visualize a large number of parameters (N>3) geometrically. The following examples are for N=2, 3, and 6. The lethality function used is a particularly simple one where the lethality =1 when the shotline point is less than a fixed distance (R) from the origin and is zero otherwise. Again, the green points are the uniformly random starting points and the red points are the estimated optimum shotline points.

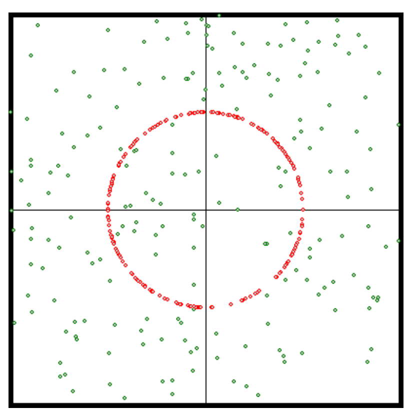

Figure 12. Set of Optimal Points for a Lethality Function with 2 Shotline parameters (cylinder)

Figure 12 is for N=2, and the optimum shotline point locus is a circle as expected.

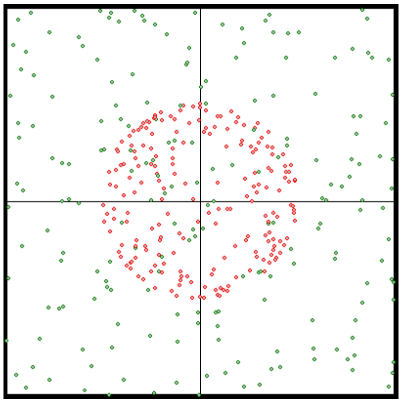

Figure 13. Set of Optimal Points for a Lethality Function with 3 Shotline parameters (sphere)

Figure 13 is for N=3. In this case the optimum shotline point locus is the surface of a sphere. The points displayed are the loci of the first two parameters. The third parameter coordinate is perpendicular to the plane of the figure and the points are actually arrayed around the surface of a sphere.

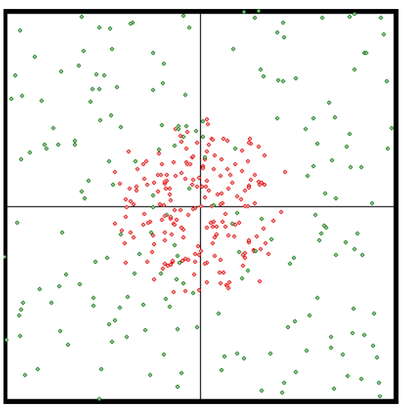

Figure 14. Set of Optimal Points for a Lethality Function with 6 Shotline parameters (hypersphere)

Figure 14 is for N=6, and the optimum shotline point locus is the surface of a hyper sphere. This was confirmed by computing the distance of the optimum points from the origin and this value was always very close to R. Again, the points displayed are the loci of the first two parameters.

Summary and conclusions

An algorithm for choosing shotline parameters for use in the experimental determination of lethal areas was presented along with a criterion for an optimal shotline. The proposed criterion for the optimal shotline is a point in the N dimensional shotline parameter space where the lethality is changing the fastest with regard to the N coordinates. The algorithm utilizes both random shotline points in the multidimensional field of concern, while also biasing the choice of the optimal shotline to conform to the criterion. Thus the entire multidimensional shotline space will be searched for instances of significant lethality, but then focus on the boundaries of the lethality space. The algorithm generates a locus of many optimal shotline points. Any one of these shotline points is suitable for the next data point. Then using the results of the test with that shotline, the model can be modified. The algorithm is applied to the latest modified model to produce another locus of optimum shotline points in the shotline parameter space. This succession of optimal data points (or N-dimensional shotlines) can be used to recursively improve the description of the lethality space using an optimizing strategy. An example appears in Reference 1, where the simulated annealing method was applied to an analytical model to develop an accurate lethality model with a relatively small number of data points required.
Even in the case where an accurate lethality model is available, the algorithm can be used to define the boundaries of the model in the multidimensional shotline parameter space, especially when N>3.

References

1. Louie, and Cooper, "Determination of Lethality Contours from Sparse Data", presented at the AIAA Missile Sciences Conference, 7-9 November 2000.